Lab on a Chip and Microfluidics

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cnrs dépasser les frontières

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Part III. Fluid transport

(pressure driven microfluidics)

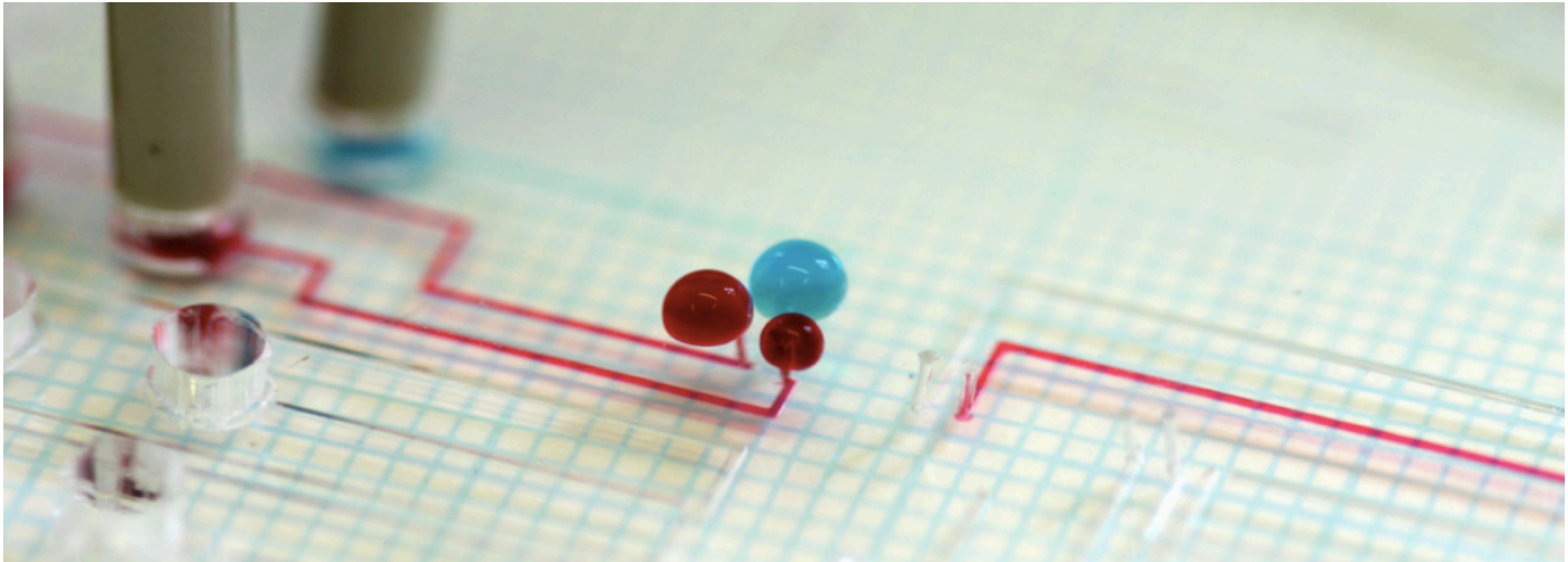
Lab On Chip fluid transport

Hydrostatic pressure

Electrokinetics

- Electro osmose (mouvement de liquides par un champ électrique)
- Electrophorèse (mouvement de particules sous l'influence d'un champ électrique)
- Dielectrophorèse

Capillary pumps

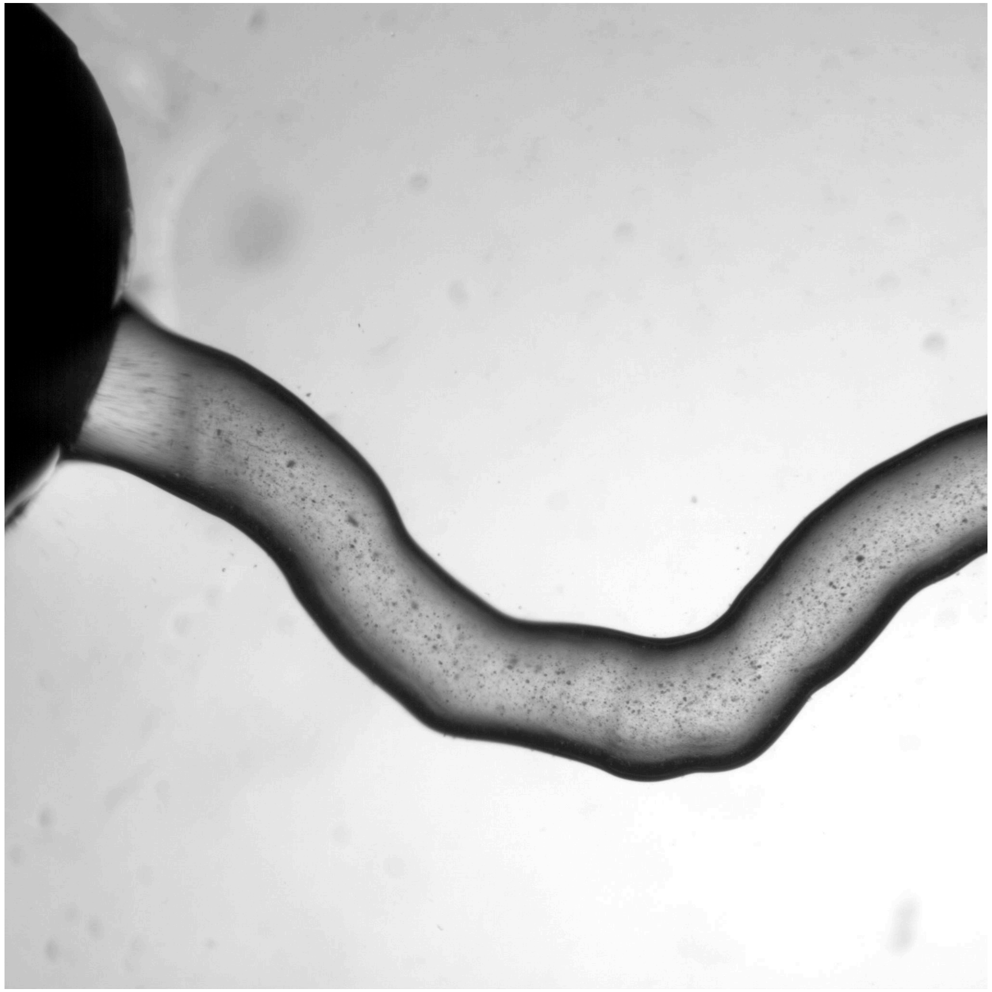


Lab On Chip fluid transport

Fluid transport : Fluid mechanics at small scale

What are the laws that govern these flows at small scale?
Pressure/flow relations?

Navier Stokes :



$$\rho\left(\frac{\partial\vec{u}}{\partial t}+\left(\vec{u}.\vec{\nabla}\right)\vec{u}\right)=\rho\vec{g}-\vec{\nabla}p+\mu\Delta\vec{u}$$

Pressure driven microfluidics

Pressure sources

Controllable
(On/Off, amplitude)

Constant
- On chip (osmotic)
- Off chip (pressure reservoir)

Hydraulic

On chip
- Thermal expansion,
- evaporation,
- micropump

Off chip
- macropump,
- syringe,
- finger push, ..

Pneumatic

On chip
- liquid evaporation, `
- micropump

Off chip
- macropump,
- syringe,
- finger push, .

Pressure driven microfluidics

Pumps

Macroscopic pumps (off chip)

Syringe pushers

Peristaltic

Membrane

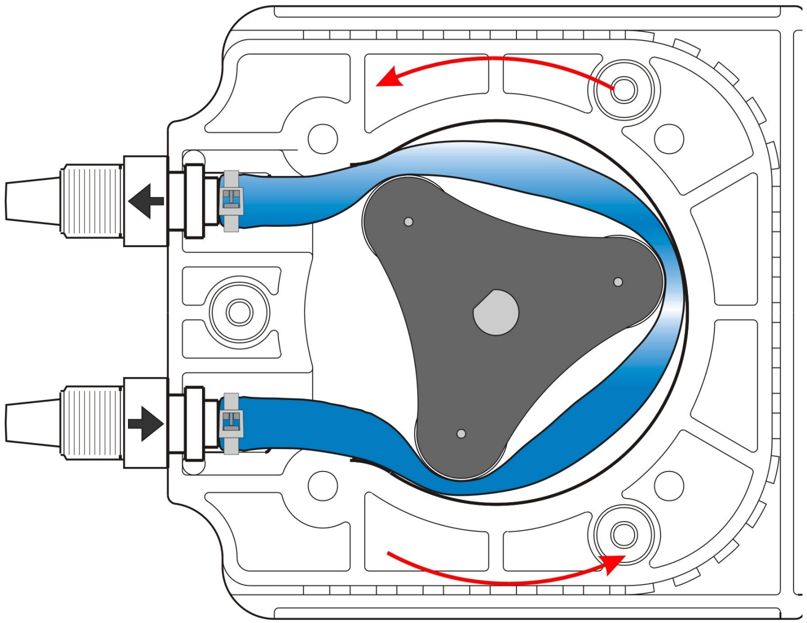


Image : blue white industries

Image : KDscientific

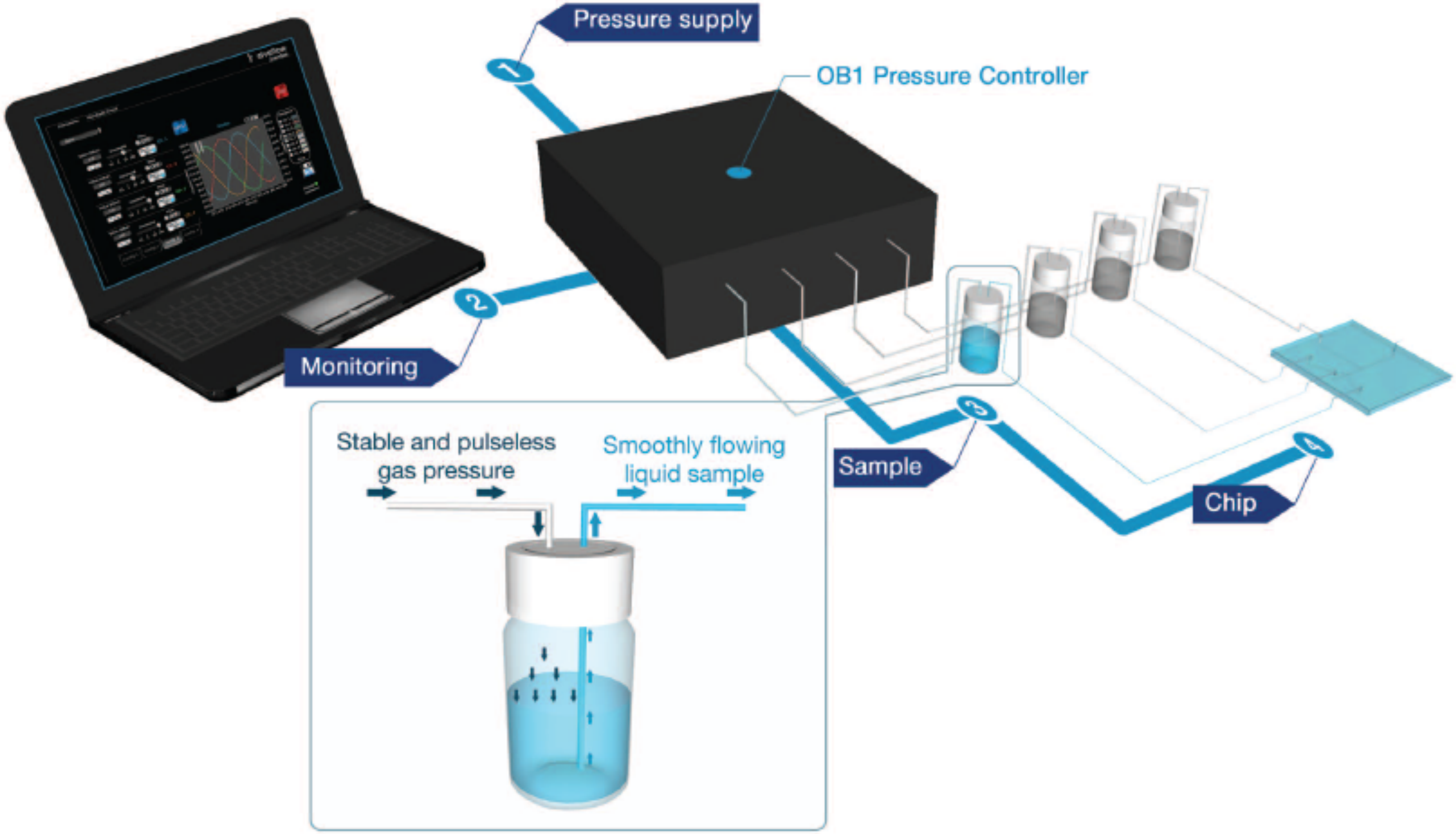
Pressure driven microfluidics

Pumps

Pressure controllers (off chips)

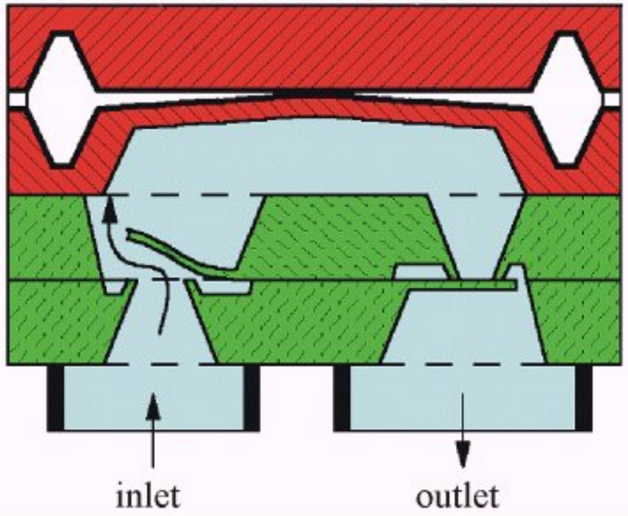
Elveflow

Fluigent

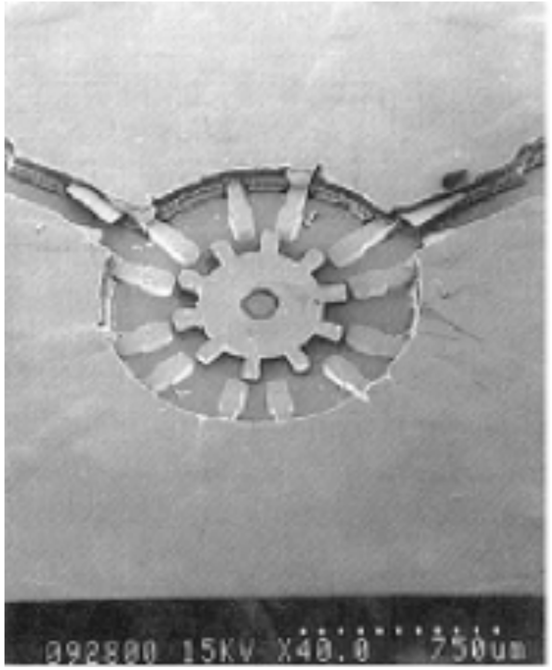


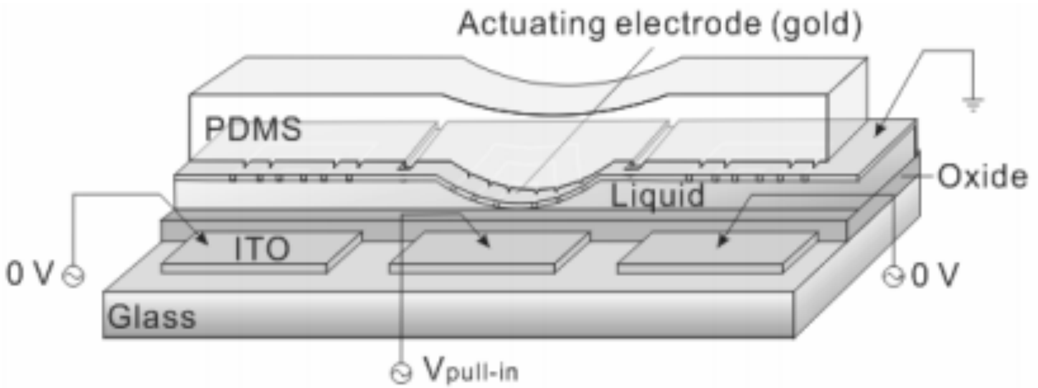
Pressure driven microfluidics

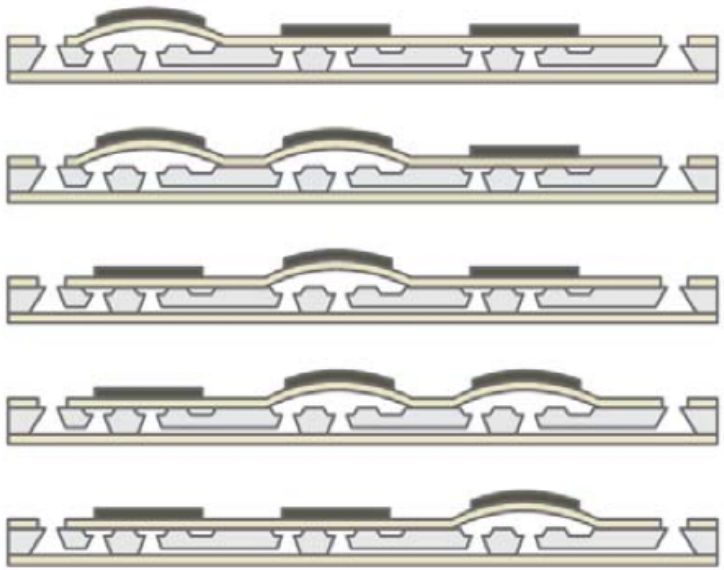
Integrated Pumps



Micropump



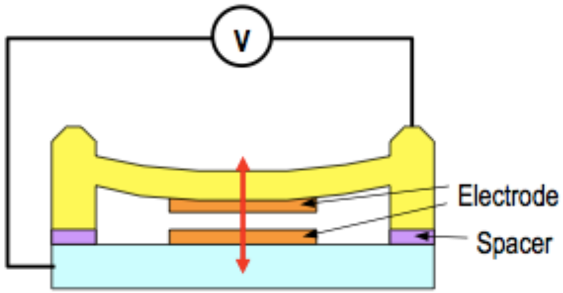


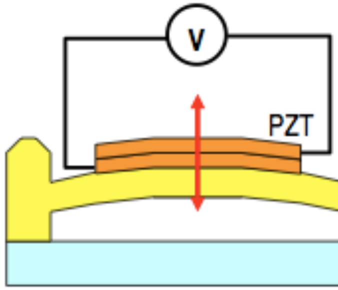


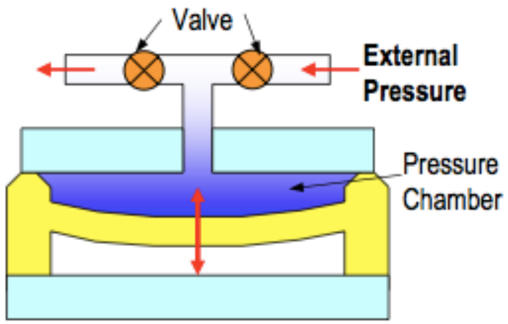
Valves

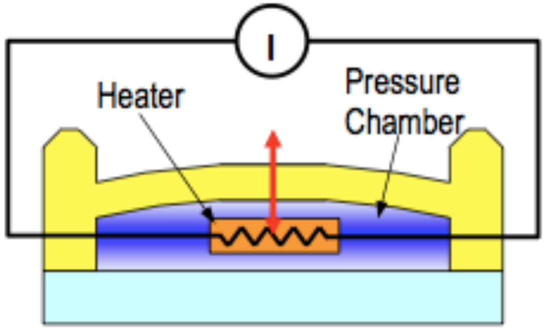
Valves : actives, passives (requires energy or not)
Normally On / Off

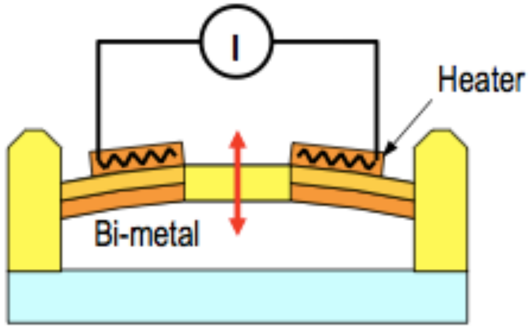
Leakage
Dead volume
Response time
Reliability
Biocompatibility
Resistance





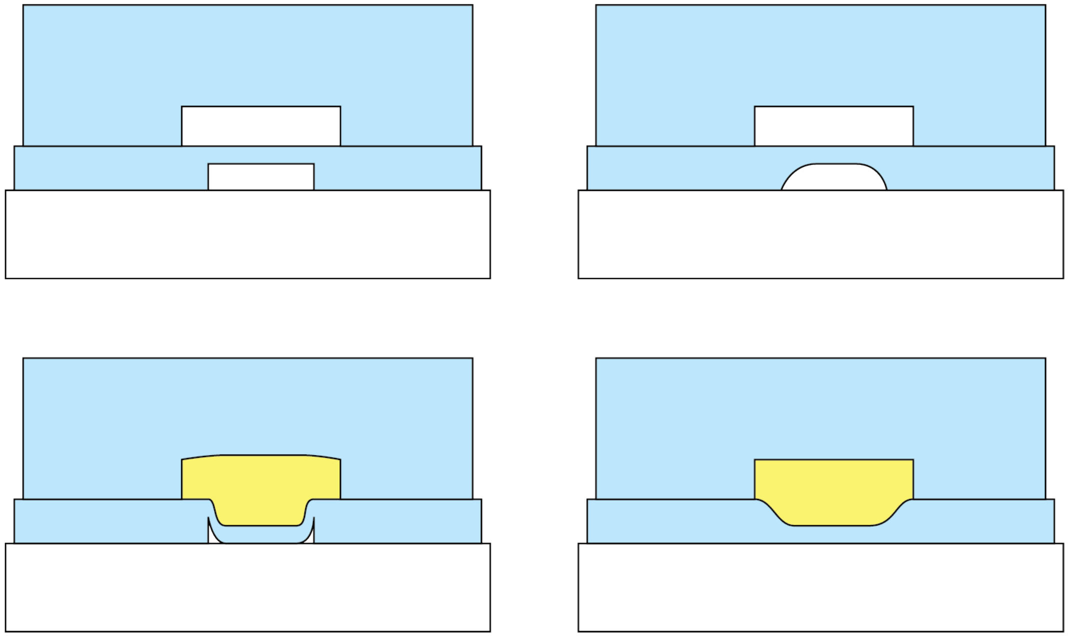


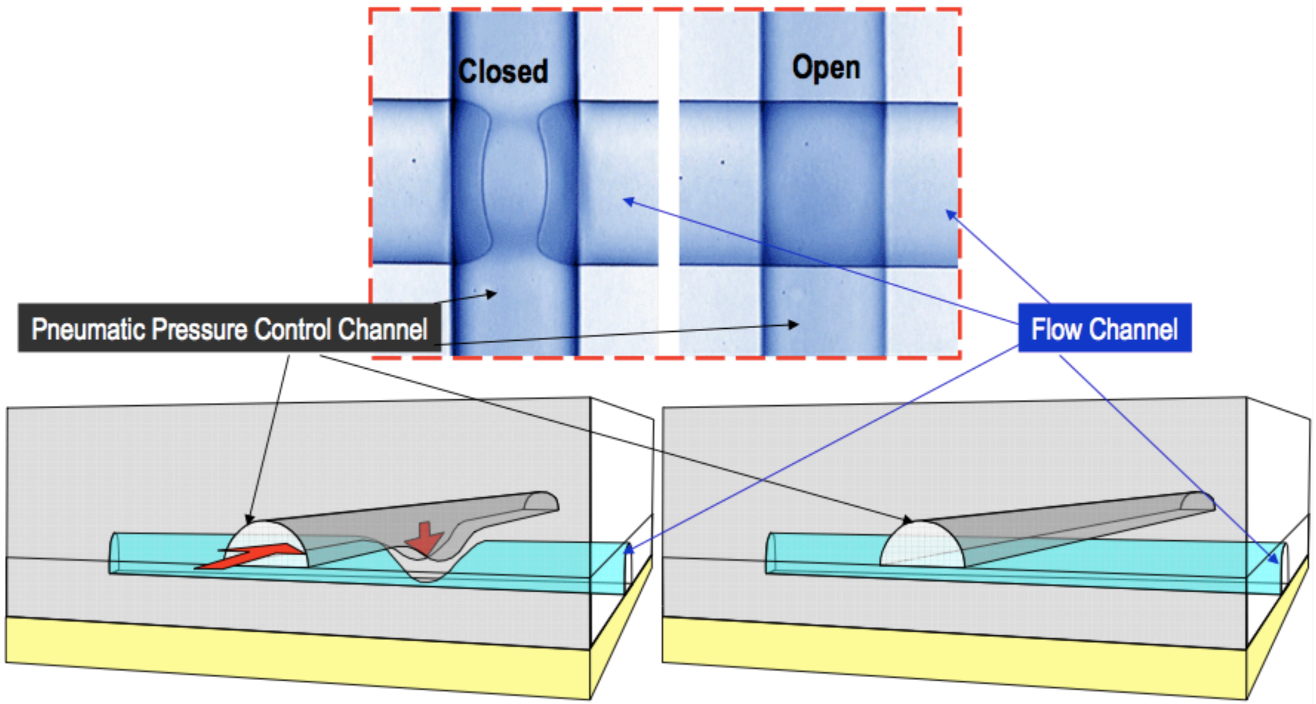




Valves

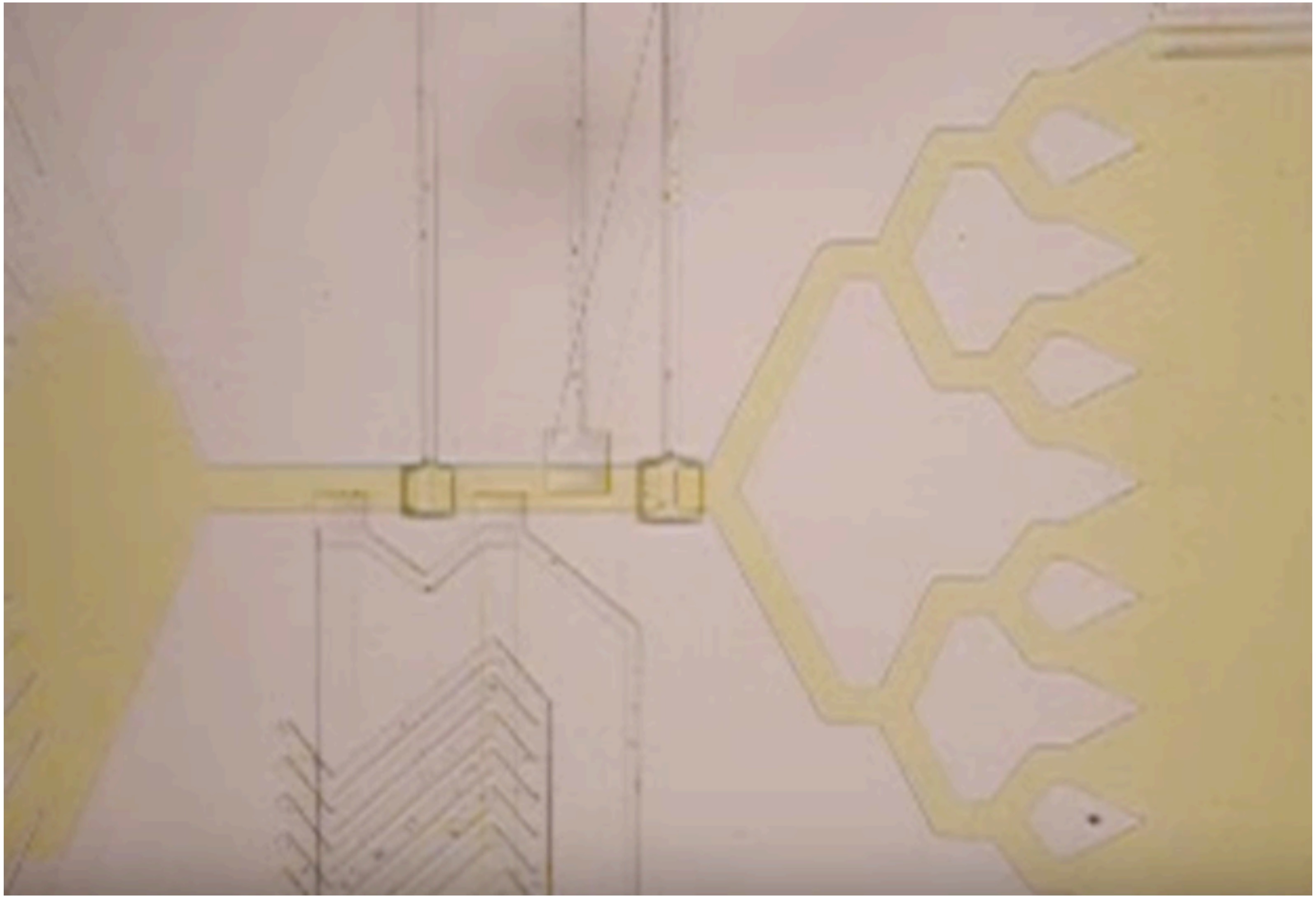
Quake valves





Valves

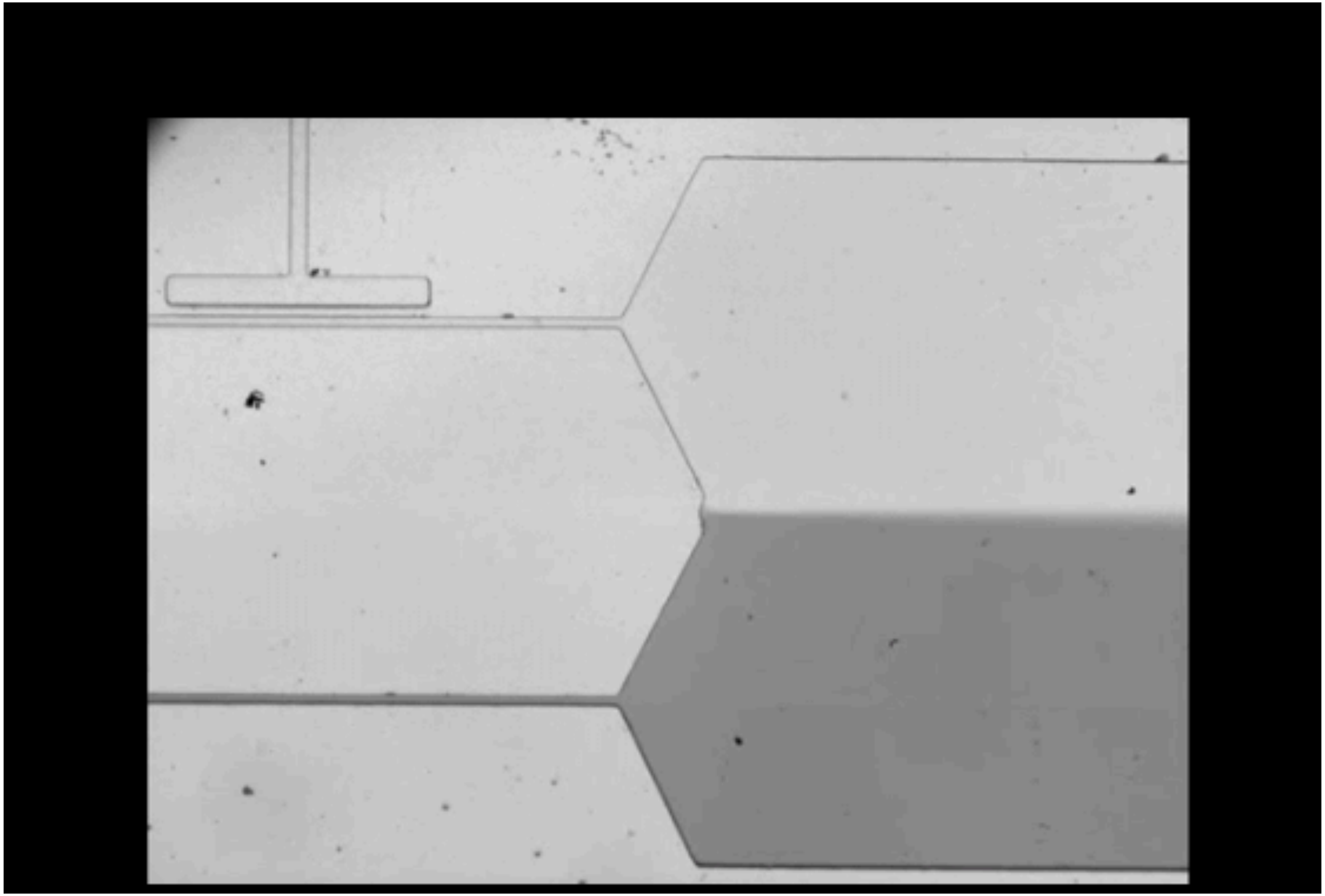
Examples



Folch lab

Valves

Examples

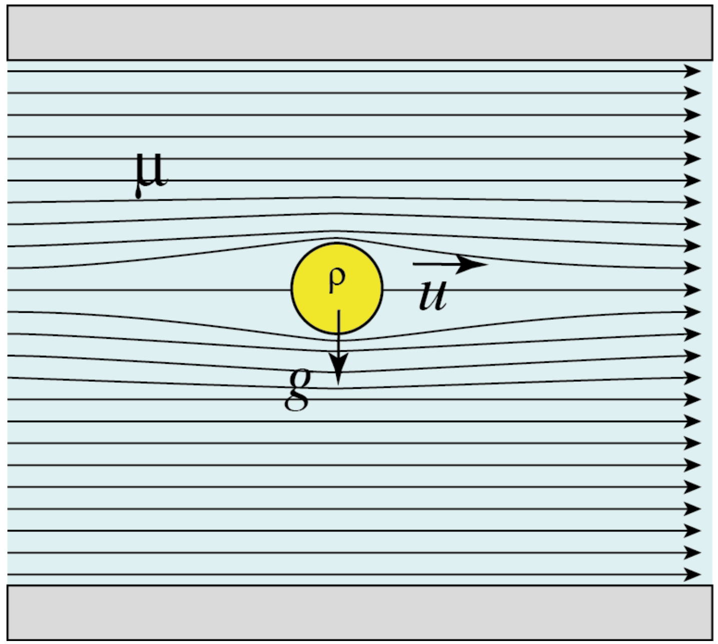


UCSF Abate Lab

Lab On Chip fluid transport

Fluid transport : Fluid mechanics at small scale

Fluidic particle



Navier Stokes equation

$$\rho\left(\frac{\partial\vec{u}}{\partial t}+\left(\vec{u}.\vec{\nabla}\right)\vec{u}\right)=\rho\vec{g}-\vec{\nabla}p+\mu\Delta\vec{u}$$

Math Reminders

Divergence and gradients

$$\vec{A} = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

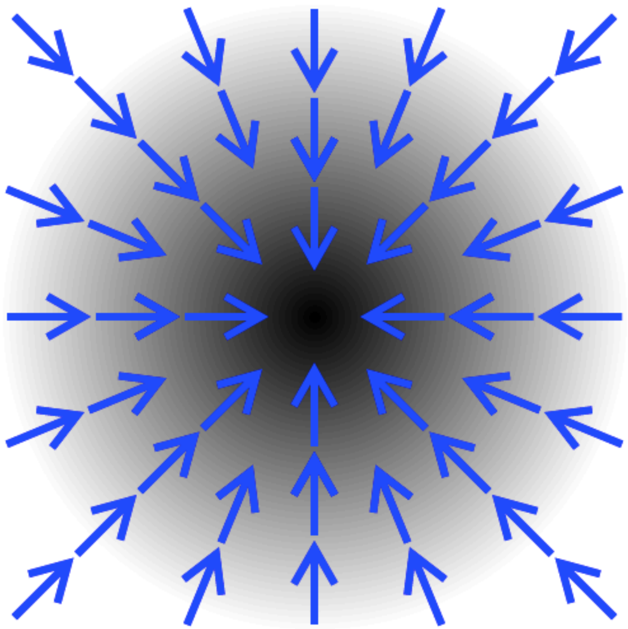
A vectorial field

$$div\vec{A} = \vec{\nabla}.\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Divergence of vector A is the scalar product of vector Nabla by vector A

$$gr\vec{a}dF = \begin{pmatrix} \frac{\partial F}{\partial x} \\ \frac{\partial F}{\partial y} \\ \frac{\partial F}{\partial z} \end{pmatrix} = \vec{\nabla}F$$

A function F



Math Reminders

Divergence and gradients

Laplacien

$$\Delta F = div(gr\vec{a}dF) = \nabla^2 F = \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2}$$

Vector Laplacien

$$\vec{\Delta}\vec{A} = \vec{\nabla}^2 \vec{A} = \begin{pmatrix} \Delta A_x \\ \Delta A_y \\ \Delta A_z \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} \\ \frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2} \\ \frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2} \end{pmatrix}$$

Navier Stokes equation

$$\rho\left(\frac{\partial\vec{u}}{\partial t}+\left(\vec{u}.\vec{\nabla}\right)\vec{u}\right)=\rho\vec{g}-\vec{\nabla}p+\mu\Delta\vec{u}$$

Mass

acceleration

weight

F pressure

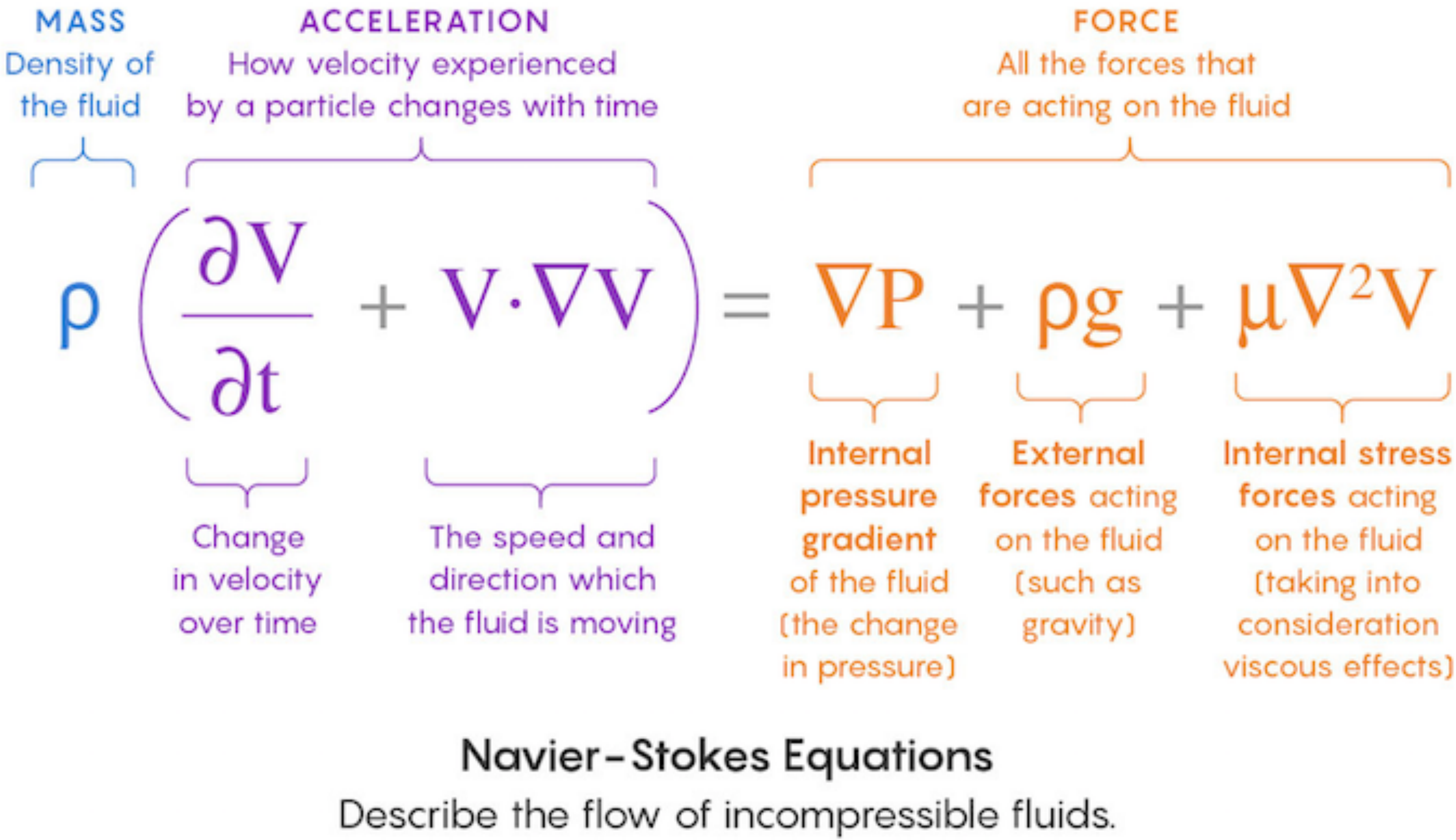
F visquous

Analog to $\sum\vec{F}=m\vec{a}$

Dimensionless numbers

Re	Reynolds	$\frac{\rho U_0 L_0}{\eta}$	inertial/viscous
Pe	Péclet	$\frac{U_0 L_0}{D}$	convection/diffusion
Ca	capillary	$\frac{\eta U_0}{\gamma}$	viscous/interfacial
Wi	Weissenberg	$\tau_p \dot{\gamma}$	polymer relaxation time/shear rate tir
De	Deborah	$\frac{\tau_p}{\tau_{\text{flow}}}$	polymer relaxation time/flow time
El	elasticity	$\frac{\tau_p \eta}{\rho h^2}$	elastic effects/inertial effects
Gr	Grashof	$\frac{\rho U_b L_0}{\eta}$	Re for buoyant flow
Ra	Rayleigh	$\frac{U_b L_0}{D}$	Pe for buoyant flow
Kn	Knudsen	$\frac{\beta}{L_0}$	slip length/macroscopic length

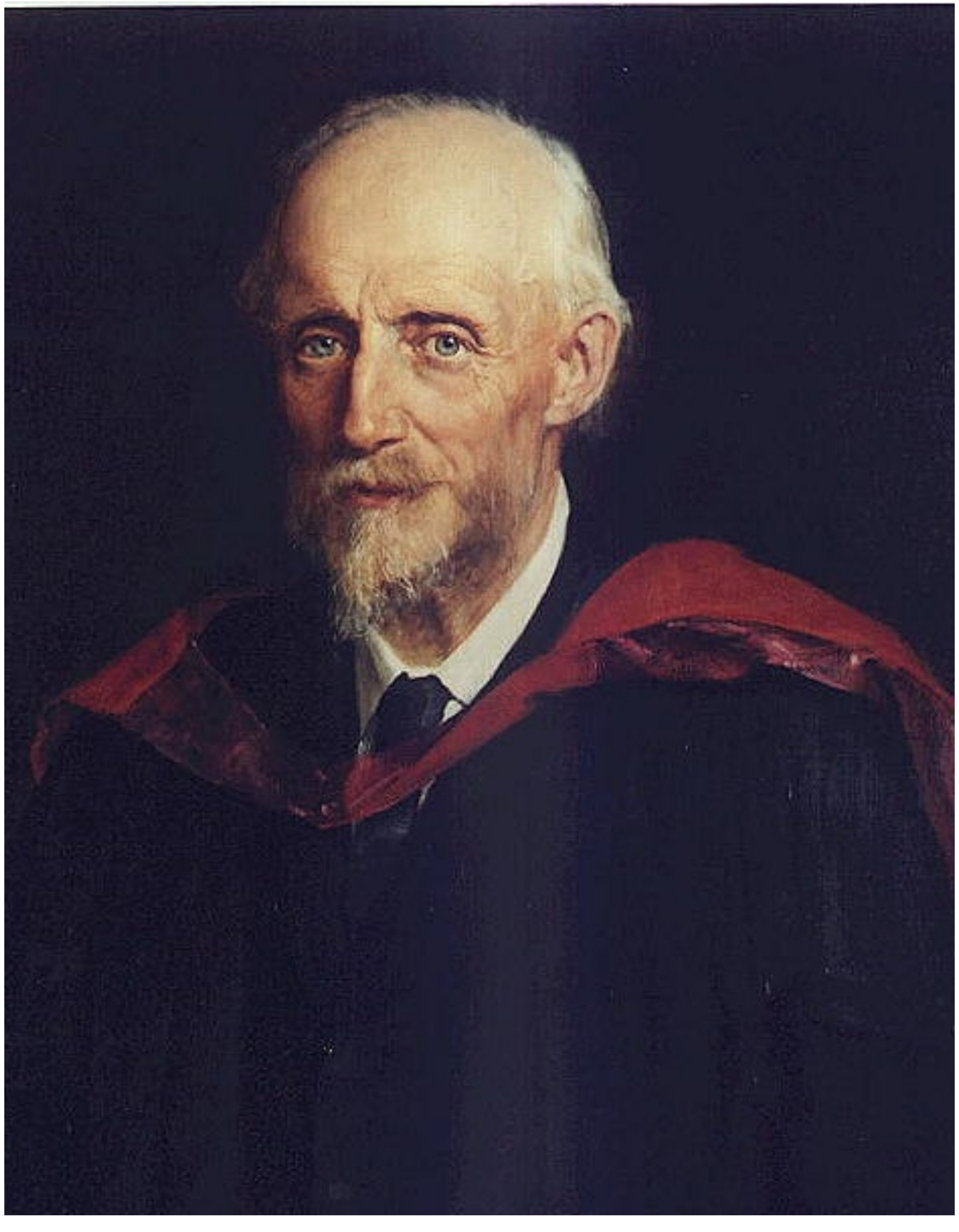
Navier Stokes equation



Reynolds Number (1883)

Osborne Reynolds

(August 23, 1842 in Belfast - February 21, 1912 in Watchet (England)) is an engineer and Irish physicist who made important contributions to hydrodynamics and fluid dynamics , most notably the introduction of number Reynolds in 1883 .



Reynolds Number (1883)

Reynolds numbers represented the ratio of momentum forces to viscous forces

It is used to label the nature of the flow: **laminar, transient or turbulent**

$$R_e = \frac{\rho v L}{\mu}$$

ρ Density
v speed
μ viscosity
L caracteristic length
(diameter of the pipe)

$R_e < 1$: Stokes regime, reversible flow, laminar, time reversal

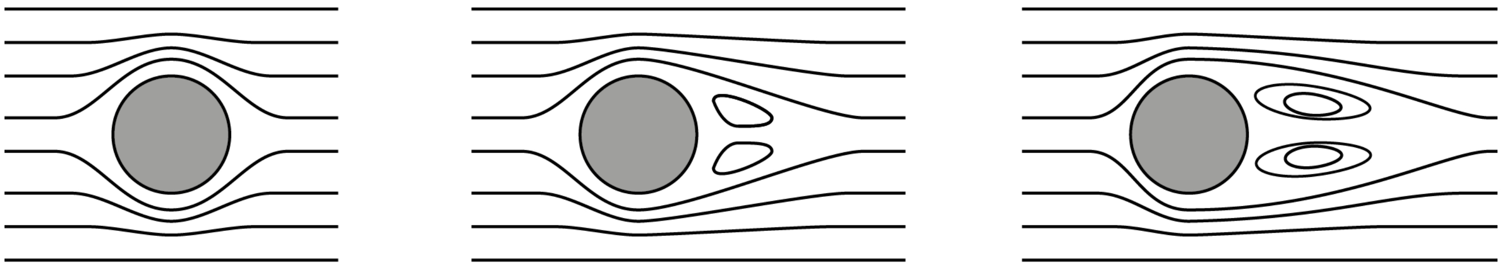
Laminar $1 < R_e < 2000$ transient

$R_e > 2000$ turbulence.

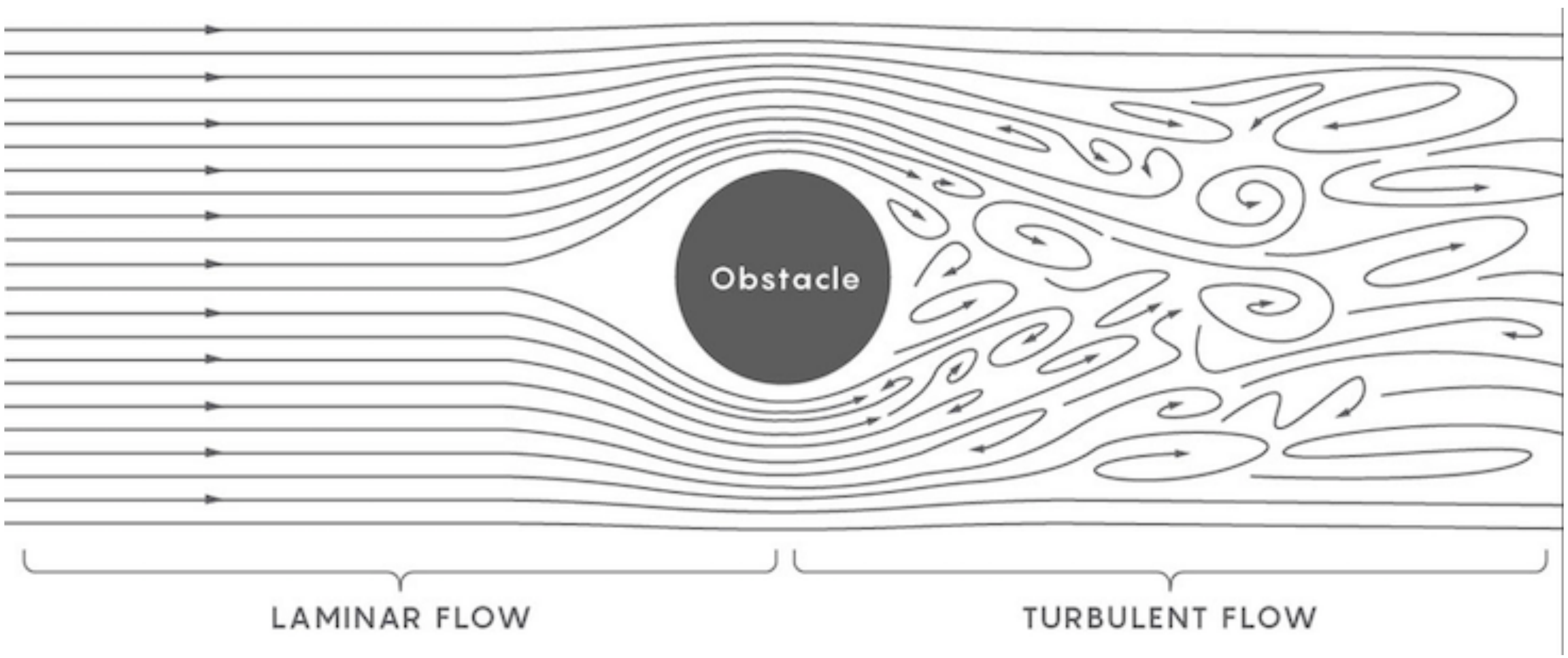
Reynolds Number

What is the direction of the flow?

Fluid flow over a cylinder

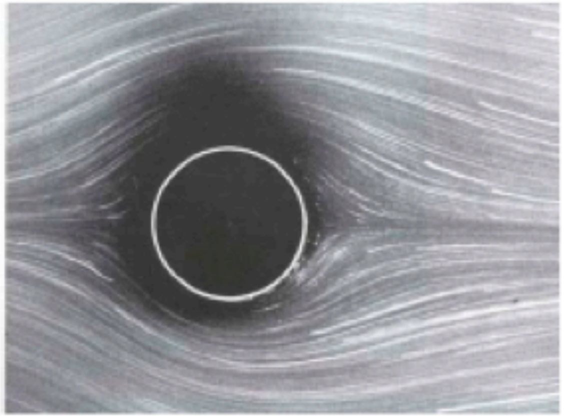


$R_e=1$ $R_e=10$ $R_e=20$

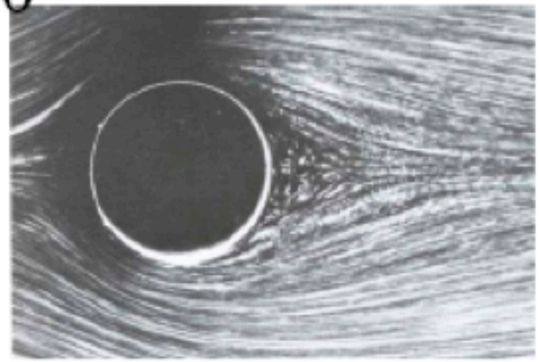


Reynolds Number

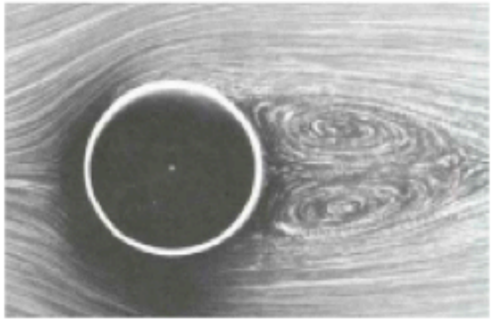
Re=1



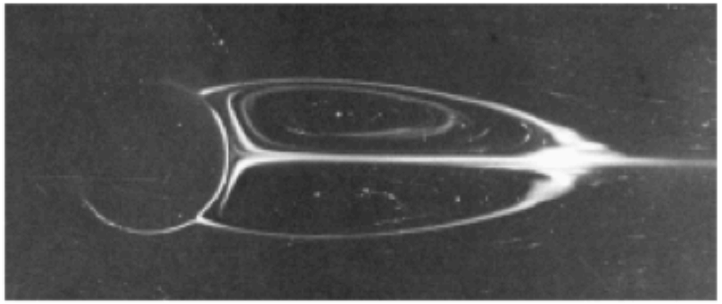
Re=10



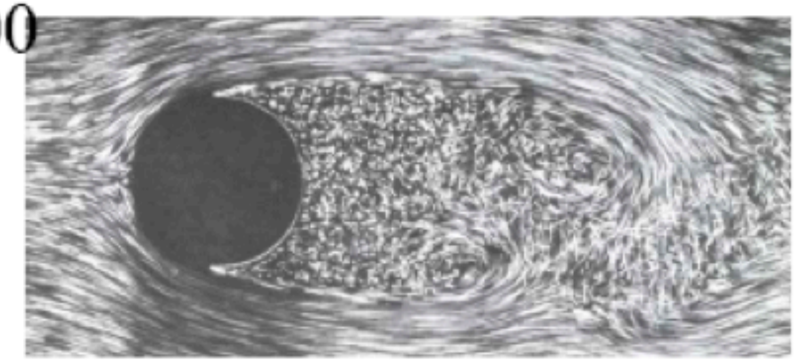
Re=26



Re=41



Re=2000



Turbulence

Reference: Van Dyke, *Album of Fluid Motion*

Turbulence

Video : projet lutetium

https://blog.espci.fr/lufr/

Reynolds Number

What about R_e in microfluidics?

Density: 10^3 Kg.m^{-3}

Viscosity : 10^{-3} Pa.s

Channel dimension : 100µm

Speed: 100µm.s^{-1}

$R_e = 10^{-2}$ **<<1**

At low Reynolds number, inertia is negligible, flows are **reversibles** and perfectly **laminar**, it is the design only that governs the flow

Poiseuille’s law

Jean Léonard Marie Poiseuille (April 22 , 1797, December 26, 1869 , Paris)
was a French physicist and physician , graduated from the Ecole Polytechnique

Movement of liquids in tubes with small diameters

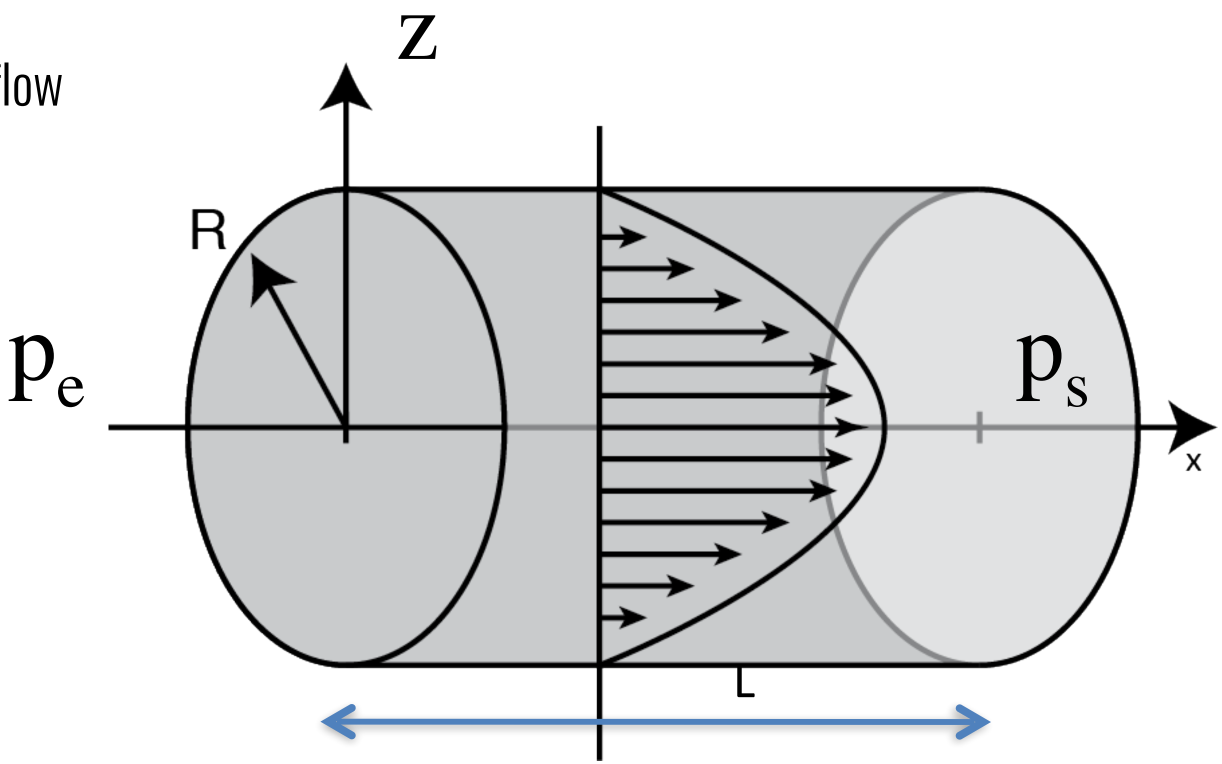
Ph.D « Recherches sur la force du cœur aortique », 1828

Also known for the Hagen–Poiseuille equation

Poiseuille's law

Problem :

- The fluid flow is parallel to the walls
- Friction at walls implies that at macroscopic scales the liquid speed is null (non sliping condition)
- Pressure doesn't change in the section of the flow
- Laminar flow $R_e < 2000$



What is the liquid velocity distibution along the section?

Solving of the problem with the Navier Stokes equation for an incompressible fluid

Navier Stokes equation for an uncompressible fluid,

$$\rho\left(\frac{\partial\vec{u}}{\partial t}+\left(\vec{u}.\vec{\nabla}\right)\vec{u}\right)=\rho\vec{g}-\vec{\nabla}p+\mu\Delta\vec{u}$$

Stokes equation In steady state, for low Reynolds number :

$$\rho\left(\cancel{\frac{\partial\vec{u}}{\partial t}}+\cancel{\left(\vec{u}.\vec{\nabla}\right)\vec{u}}\right)=\rho\vec{g}-\vec{\nabla}p+\mu\Delta\vec{u}$$

$$\rho\vec{g}-\vec{\nabla}p+\mu\Delta\vec{u}=0$$

Poiseuille flow

Neglecting weigth (microfluidics)

$$\vec{\nabla} p = \mu \Delta \vec{u}$$

If the section is small compared with length, Pressure P is only function of x and velocity distribution is function of z
Equation becomes :

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u(z)}{\partial z^2}$$

Poiseuille flow

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u(z)}{\partial z^2} = cte = \alpha$$

$$p(x) = \alpha x + \beta \qquad\qquad u(z) = \frac{\alpha z^2}{\mu} + \gamma z + \delta$$

Boundary conditions

Pressure

$$p(0) = p_e$$

$$p(L) = p_s$$

Velocity

$$u(r) = 0$$

$$u(-r) = 0$$

$$P(x) = \frac{p_s - p_e}{L} x + p_e$$

$$u(z) = \frac{p_s - p_e}{4\mu L} r^2 \left(1 - \left(\frac{z}{r}\right)^2\right)$$

Poiseuille flow

Maximum velocity

$$v(z) = v_{\max}\left(1 - \frac{z^2}{r^2}\right)$$

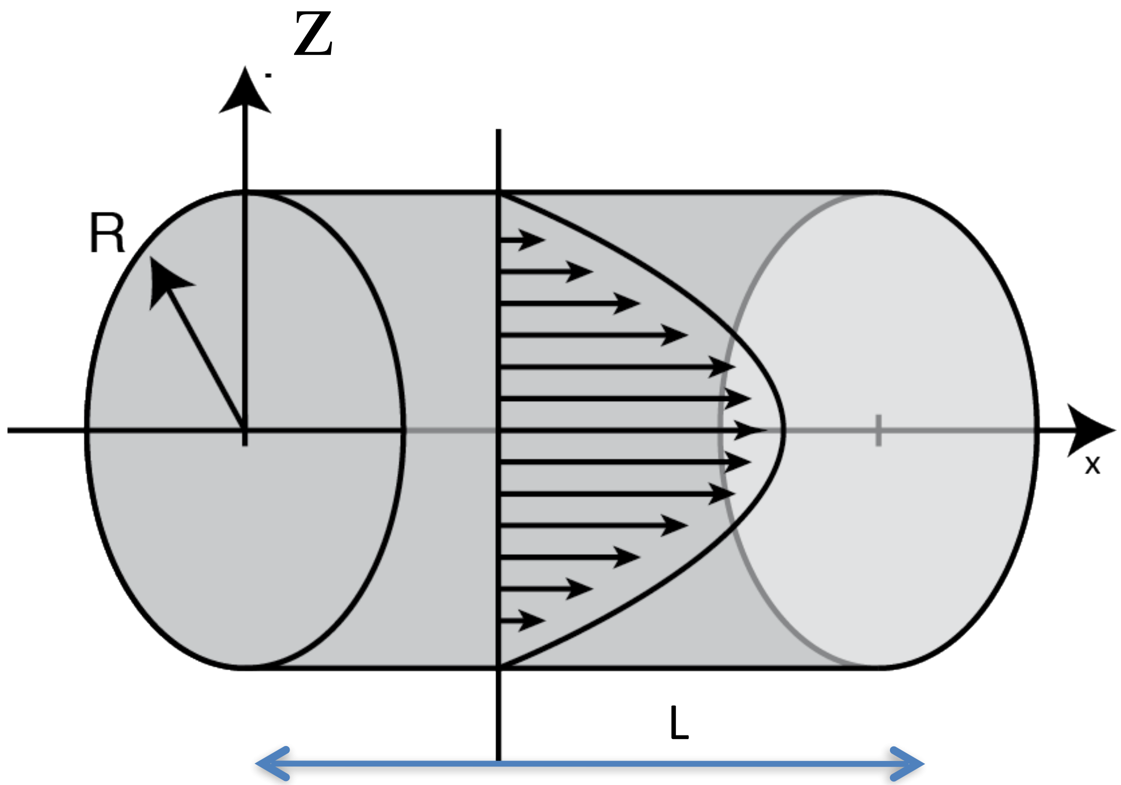
Mean velocity

$$v_m = \frac{v_{\max}}{2} = \frac{R^2}{8\mu}\frac{dp}{dx} = \frac{Q}{S}$$

$$\Delta p = 8\mu\frac{L}{Sr^2}Q = 8\mu\frac{L}{\pi r^4}Q$$

$$\Delta p = R_{hydro}Q$$

$$U = R_{elec}I$$



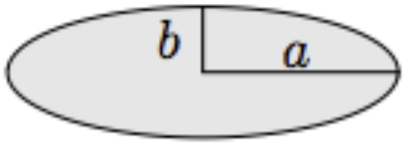
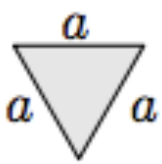
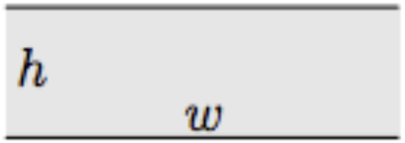
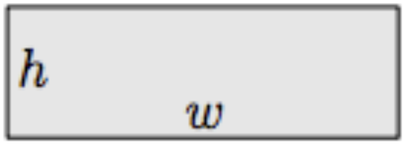
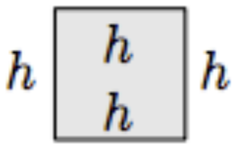
$$R_{hydro} \propto L$$

$$R_{hydro} \propto r^{-4}$$

Hydro resistance increases when the section decreases **(power 4)**

Poiseuille flow

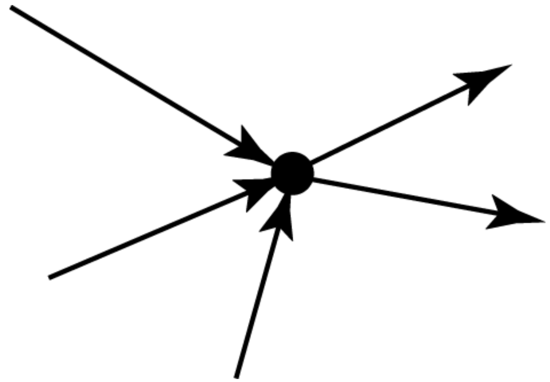
Hydro resistance for different sections

circle		$\frac{8}{\pi}\eta L\frac{1}{a^4}$
ellipse		$\frac{4}{\pi}\eta L\frac{1+(b/a)^2}{(b/a)^3}\frac{1}{a^4}$
triangle		$\frac{320}{\sqrt{3}}\eta L\frac{1}{a^4}$
two plates		$12\,\eta L\frac{1}{h^3 w}$
rectangle		$\frac{12\,\eta L}{1-0.63(h/w)}\frac{1}{h^3 w}$
square		$\frac{12\,\eta L}{1-0.917\times 0.63}\frac{1}{h^4}$

Microfluidic Network

The sum of flow rate is conserved at a junction
Equivalent to Kirchhoff's circuit laws « The current entering any junction is equal to the current leaving that junction »

$$\sum_i Q_e = \sum_i Q_s$$

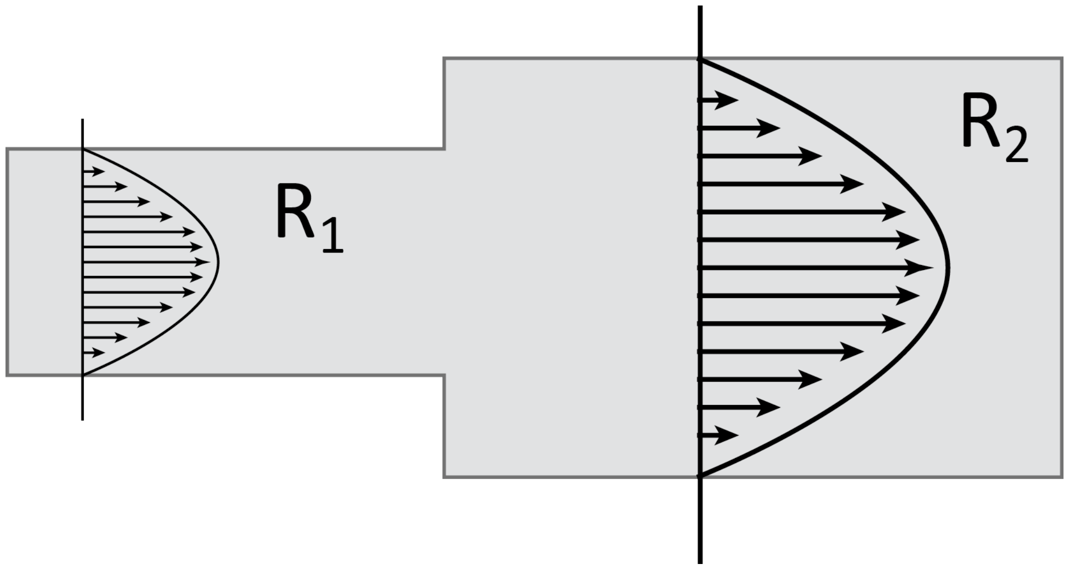


$$\sum_{i=2}^{n} \left(P_i - P_{i-1}\right) = 0$$

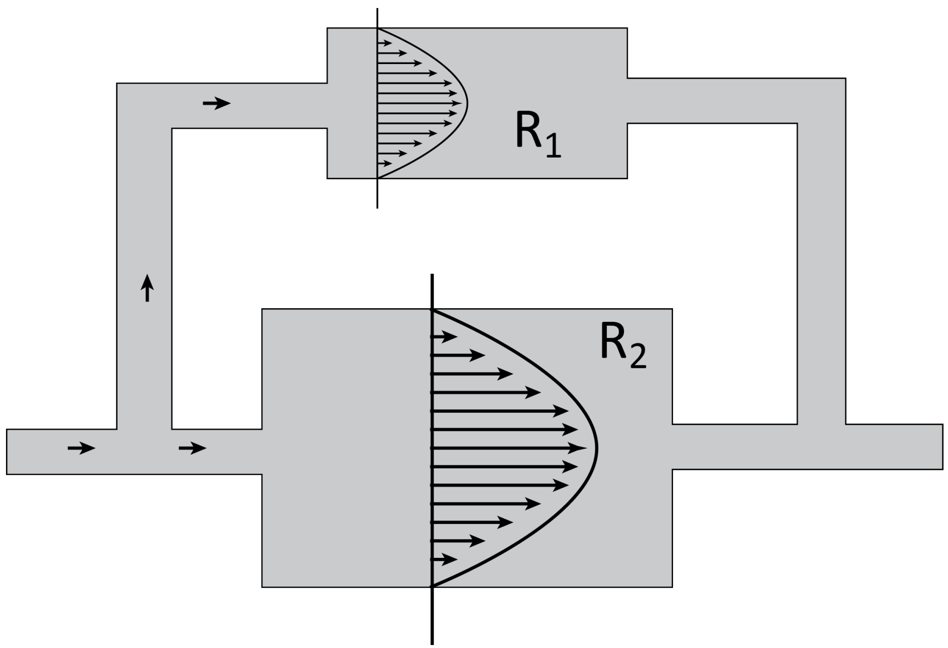
The sum of all the pressure drop around a loop is equal to zero

Microfluidic Network

$$R_{eq} = R_1 + R_2$$



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$



Microfluidic Network

Electric Circuit	Hydraulic Circuit
U Battery	**P** Pump (pressure controller)
I Current generator	**Q** Pump (syringe pusher)
Δ**U** Potential difference	Δ**P** pressure difference
I Current	**Q** flow rate
i current density	**v** flow speed
R Resistance	$\mathbf{R_h}$ hydro resistance
C Capacitance	$\mathbf{C_h}$ compliance
L Inductance	Fluid Inertia (negligible)
Power ΔUI	power ΔPQ

Microfluidic Network : application

Stokes trap for multiplexed particle manipulation and assembly using fluidics

Anish Shenoya, Christopher V. Raob, and Charles M. Schroederb,

PNAS, vol. 113 no. 15

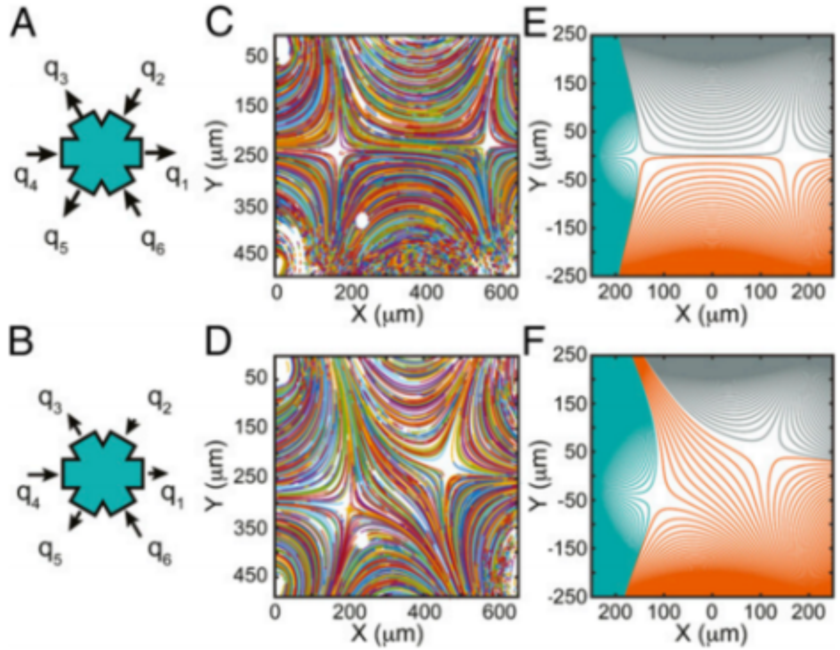
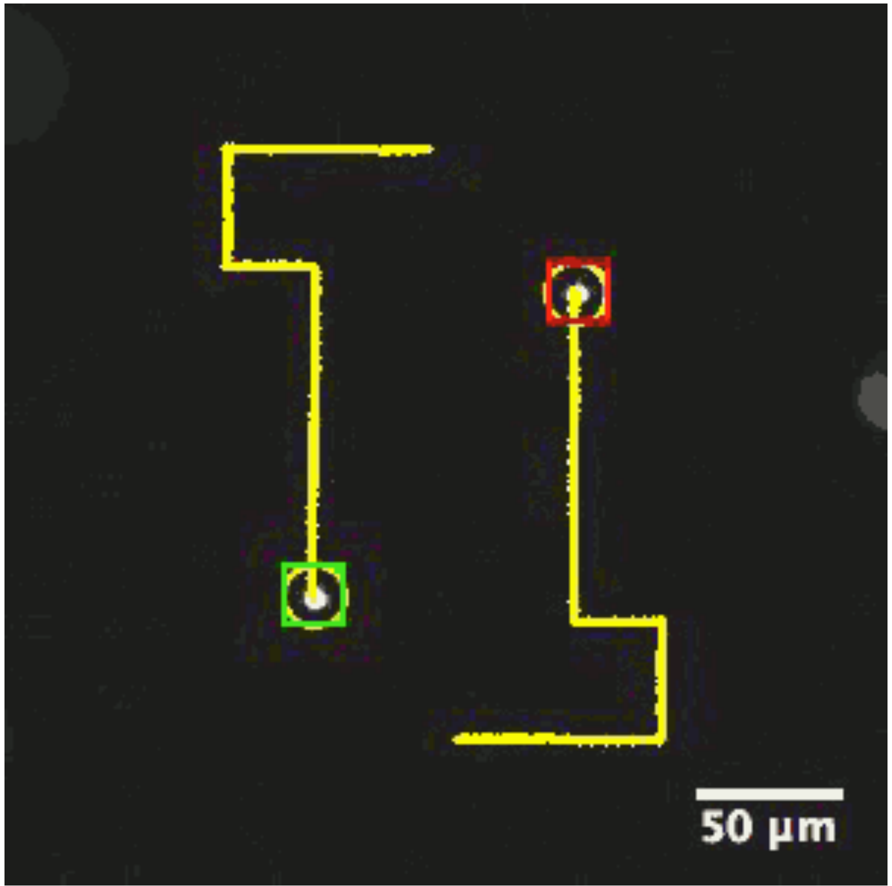
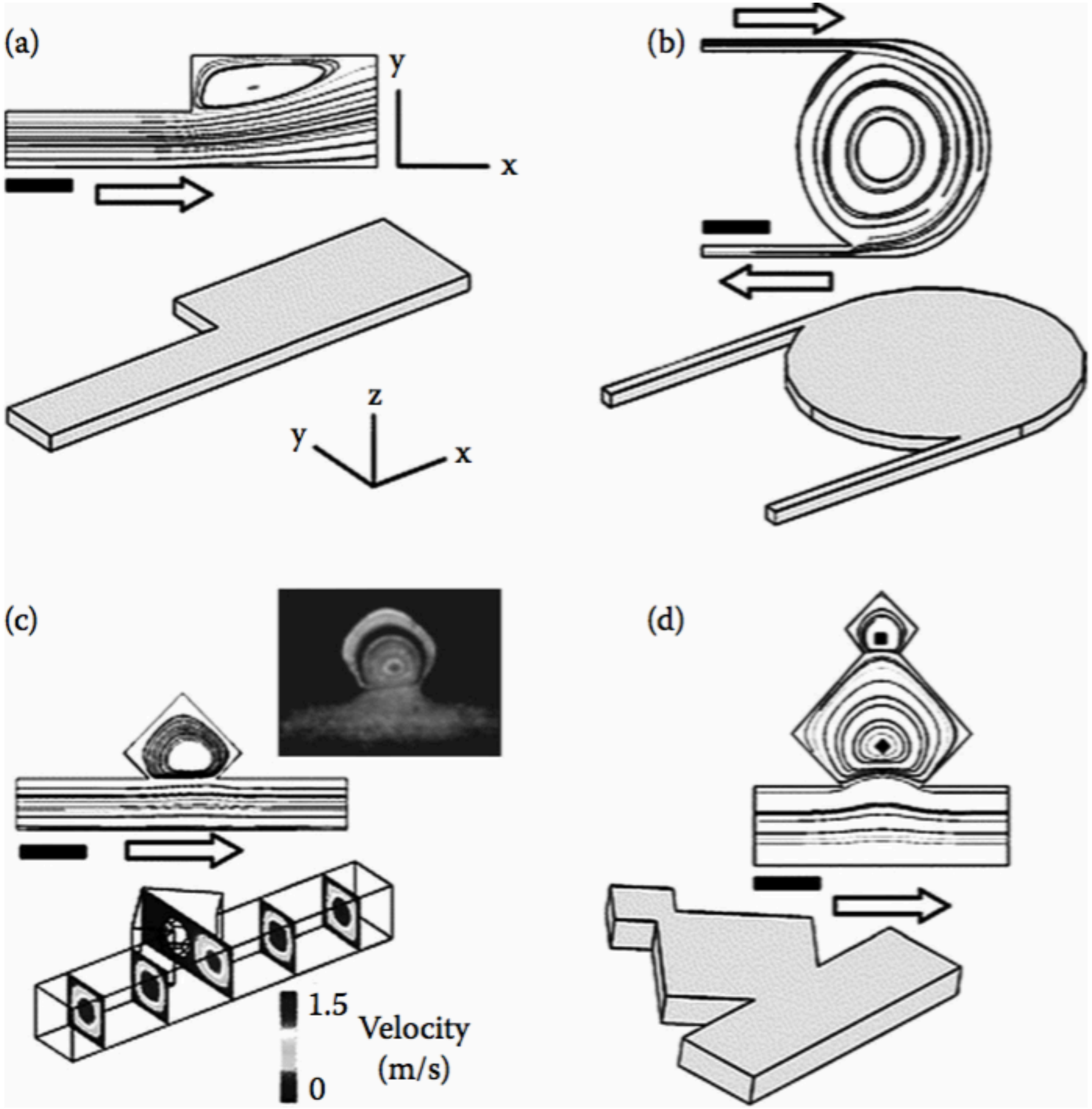


Fig. 2. Streamline topologies in a six-channel microdevice from experiments and computation. (*A* and *B*) Schematic of the relative magnitude and direction of the flow rates for generating the streamline topologies in the figures below. Arrows pointing inwards represent flow entering the device, and arrows pointing outwards represent flow exiting the device. The size of the arrows signifies the relative magnitude of the flow rates. (*C*) Experimental streamlines showing the linked-arms topology, generated when the flow rates have a specific symmetry. Two stagnation points are clearly visible. (*D*) Experimental streamlines showing the non–linked-arms topology, generated if the symmetry in *C* is broken. (*E* and *F*) Streamline topologies obtained from numerical solution of Eq. **2**. For display, streamlines emanating from inlet channels are plotted using distinct colors.



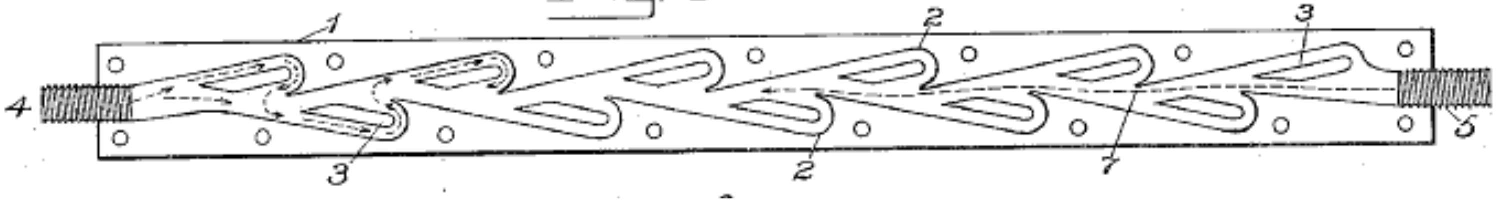
Using a six-channel microfluidic device, scientists can alter the flow in the device in such a way that they trap and manipulate two particles at the same time.

Recirculation



Tesla valve

Fig. 1



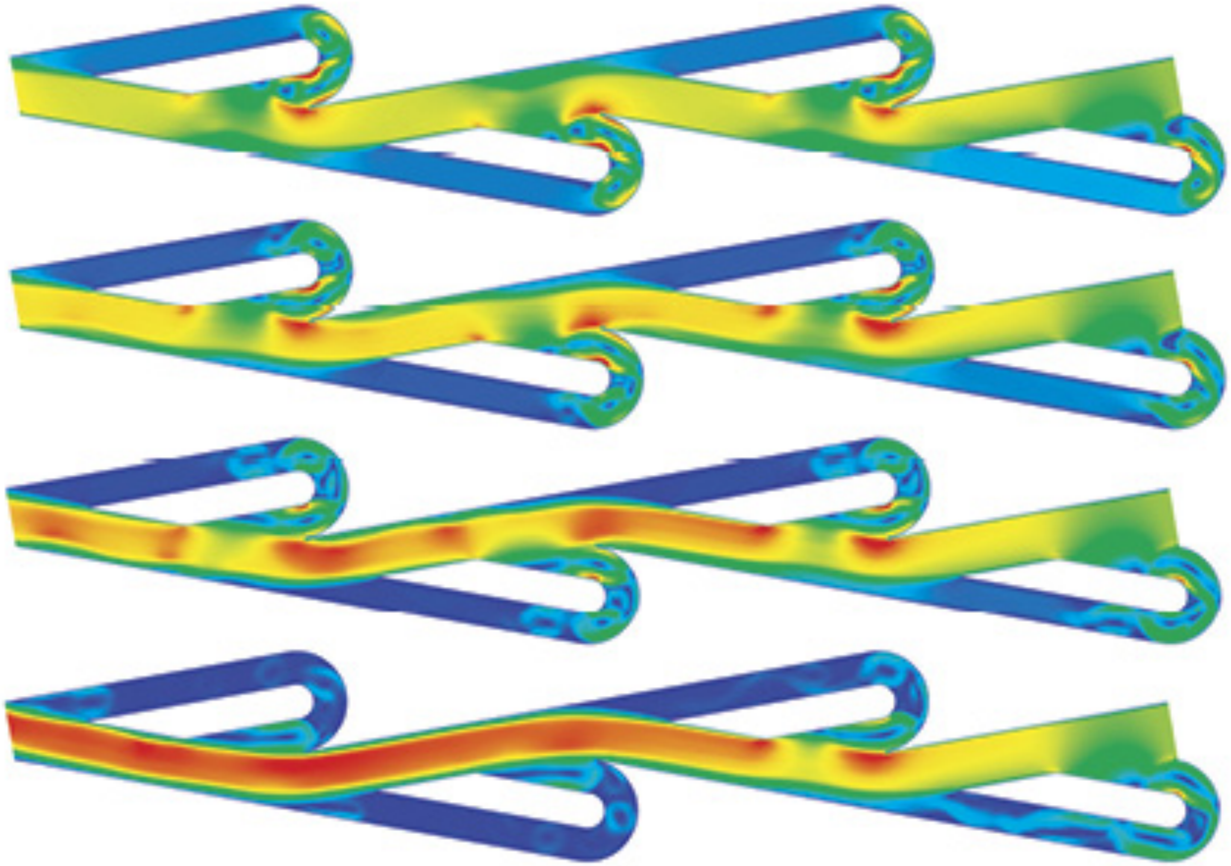
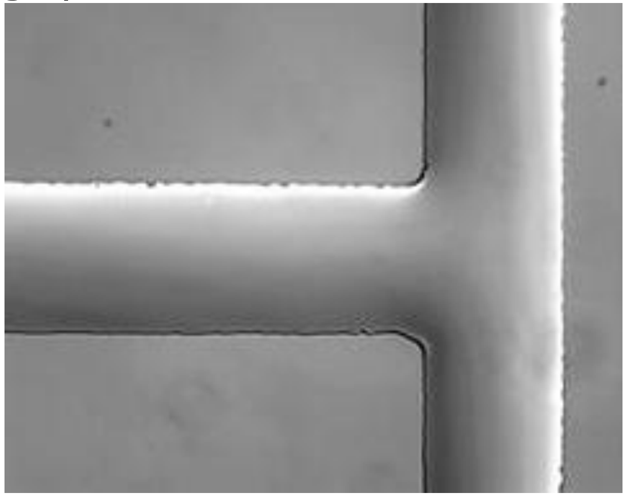
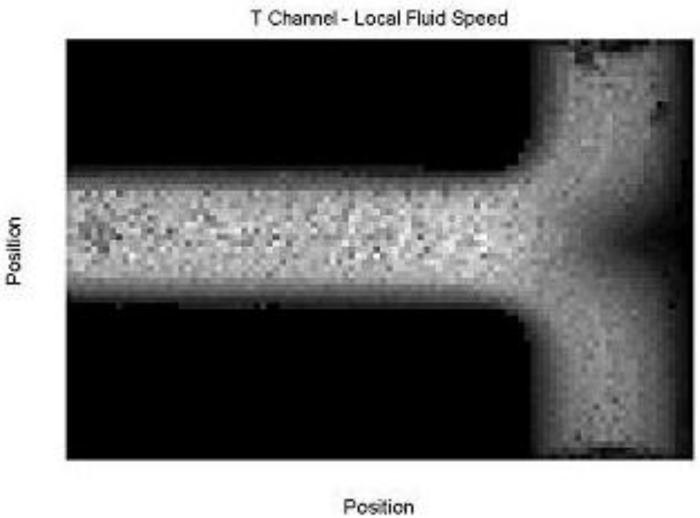


Fig. 3: Flow development in the unimpeded direction

Flow visualization

- Staining
- Micro particle image velocimetry
- Doppler
- holography



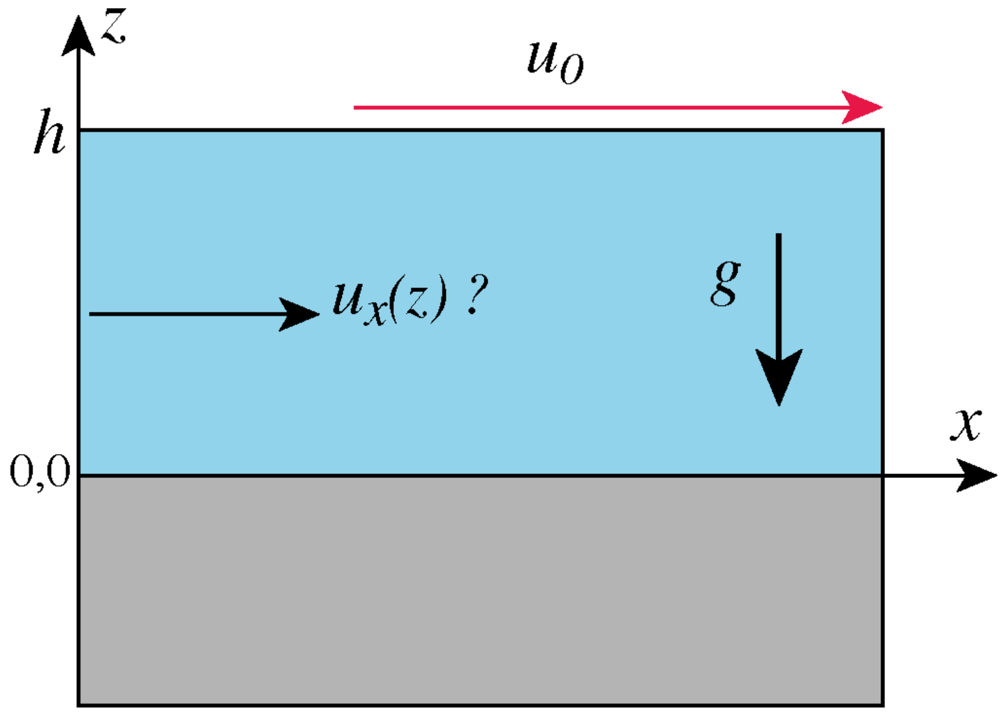


Couette flow

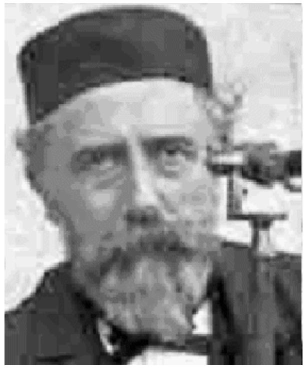
A film of water on a flat substrate

Initial velocity $u_x(h)=v_0$

What is the velocity distribution $u_x(z)$?



Maurice Marie Alfred Couette, Born january 9, 1858 in Tours, France, and died August 18, 1943, is a French Physicist whose work focused mainly on fluid mechanics and especially on rheology . He defended his thesis for the doctorate of science on friction in liquids in the laboratory of physical research of the Faculty of Sciences of Paris . His name is primarily associated with Couette flow but also for the cylinder viscometer that bears his name .

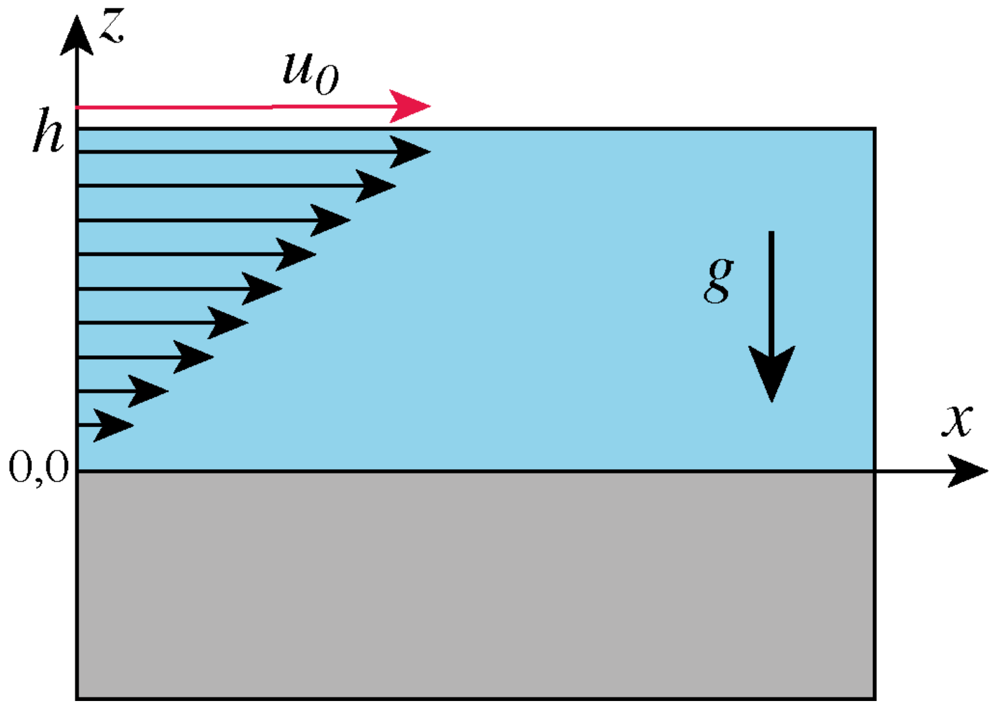


Couette flow

$$\rho\left(\frac{\partial\vec{u}}{\partial t}+\left(\vec{u}.\vec{\nabla}\right)\vec{u}\right)=\rho\vec{g}-\vec{\nabla}p+\mu\Delta\vec{u}$$

$$\left(\vec{u}.\vec{\nabla}\right)\vec{u}=0$$

Steady state Navier flow



$$\rho\vec{g}-\vec{\nabla}p+\mu\Delta\vec{u}=0$$

$$\frac{\partial^2}{\partial z^2}u_x(z)=0$$

$$u_x(z)=az+b$$

Boundary conditions

$$u_x\left(0\right)=0$$

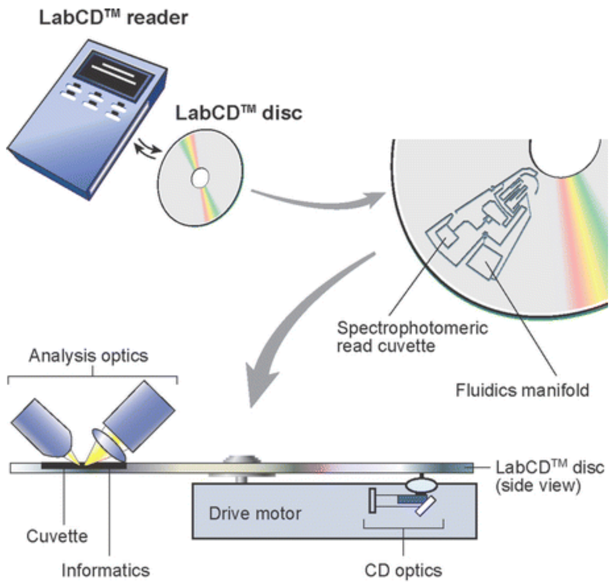
$$u_x\left(h\right)=u_0$$

$$u_x(z)=\frac{u_0 z}{h}$$

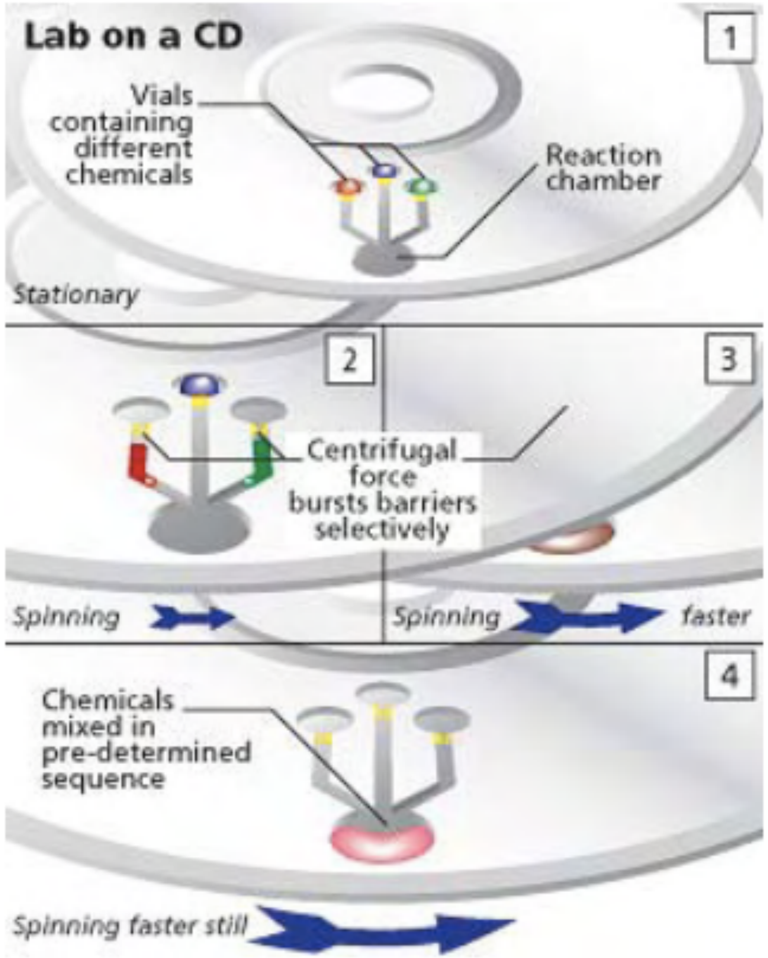
Centrifuge microfluidics : Lab On a CD

Microfluidic circuits printed on a Compact disc

Use of centrifuge force induced by the rotation to move liquids



Madou M, et al. 2006. Annu. Rev. Biomed. Eng. 8:601–28



Lab on a Chip and Microfluidics

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